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ON CONDITIONS IMPOSED ON SECTIONS OF A KRIPKE STRUCTURE THAT SIMULATE THE FUNCTIONING OF THE COMPOUND COMPONENTS ALLOCATED IN DETAILED PETRI NET OF PARALLEL DISTRIBUTED SYSTEM FOR VERIFICATION OF ACCURACY OF TEMPORAL LOGIC FORMULAE

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Abstract. Kripke structures of detailed Petri net and its component Petri net of parallel distributed system are investigated. Necessary and sufficient conditions are established for checking the validity of formulas of temporal CTL-logic on reduced Kripke structure — Kripke structure of component Petri net.

1. Introduction

Method for model checking [1, 2, 3] is one of the most convenient methods for verifying complex real systems. This method involves building of the model of the studied system and checking the system under consideration for possession of the necessary property. For this purpose, for the system researched, one type of apparatus is built — its Kripke structure. The property required, for which the system undergoes checking, is written in terms of temporal logic [1, 2, 4], then the accuracy of this formula is determined on the Kripke structure built. For systems with concurrency, such structures may have a huge number of states, and thus verification of such systems is very difficult if possible. The problem of building an adequate Kripke structures (models corresponding to a given analyzed system), the dimensions of which are suitable for verification, can be solved, for example, by reduction of the originally built models [3].

In [5, 6] as necessary Kripke structure, a reduced Kripke structure K_{CN} of investigated parallel distributed systems was considered, obtained as Kripke structure of component Petri net (CN-Net) of investigated parallel distributed system [7, 8]. It is shown that Kripke structures K_N and K_{CN} accordingly of detailed Petri net and its component Petri net system under investigation are homomorphic [6] and bisimular [5]. Homomorphism stated of obtained Kripke structures of investigated system allowed to obtain algorithm of accuracy verification formulas of temporal CTL-logic, specifying system property, on the reduced Kripke model — Kripke structure component of Petri net of the system being analyzed.

The purpose of this work is to establish uniform conditions which should have all parts of Kripke structure K_N that simulate the functioning of the compound components of the component CN Petri net of the investigated parallel distributed system, at the proof of the accuracy of CTL-logic formulas, used algorithm [6] to check CTL-formulas on the model K_N by checking accuracy of this formula on the reduced model.

The presence of such common conditions will significantly improve the previously proposed algorithm.

2. Statement of the problem

Studies, carried out in [5, 6], allow as a reduced Kripke structure to use Kripke structure K_{CN} of component CN Petri net of the system under consideration. CN component Petri net is an adequate model of the researched system and has significantly smaller dimensions than the original detailed Petri net N of the studied system. The CN net is based on the N net by separating compound components: component-places C_p and component-transitions C_t. Consequently, Kripke structure K_{CN} of component Petri net has much smaller size than the original Kripke structure of detailed Petri net N.

Set of states of Kripke structure K_N is partitioned into disjoint equivalence classes by the relation of component χ_1 [9], each class includes states for which the following conditions are true:

1) each state of Kripke structure K_N is in relation χ_1 with itself;

2) two states of Kripke structures K_N are in relation χ_1 if they are states of one section of Kripke structure K_N, which reflects the dynamics of the functioning of the respective compound components allocated in the net. If the state of a Kripke structure K_N is not a state of any section of the Kripke structure of net N, reflecting the dynamics of functioning of compound component allocated in N, then this state represents by itself an equivalence class — the unit class.

In [5], the following rules for the interconnected verification for models

$$K_N = (G, G_0, R, f) \quad \text{and} \quad K_{CN} = (G', G'_0, R', f')$$

are established:

1) for a single equivalence class and atomic statement $p \in P \quad f(g) = f'(h(g))$ is fulfilled, where g is any state of G, $h(g) = g'$, $g' \in G'$, h — homomorphism of these models K_N and K_{CN}, as a result of which each section of the structure K_N, reflecting the dynamics of functioning of compound components, is encapsulated in one state (state-encapsulant) in the model K_{CN};

2) for non-single equivalence class and atomic statement $p \in P$ are true in each state of section of Kripke structure K_N, reflecting the dynamics of the operation of

corresponding compound component, allocated in the net N, we have the accuracy of this atomic statement in the corresponding state-encapsulant of structure K_{CN};

3) if an atomic statement $p \in P$ is true in a state-encapsulant of structure K_{CN}, then it sufficient to carry out check on the validity of the atomic statements in the structure K_N in states of only one of the identical section of structure K_N, reflecting the dynamics of the operation of the same compound component;

4) if the formula φ of CTL-logic is not performed on the reduced structure K_{CN}, this formula is not met either on detailed models of the original detailed Petri net.

In [6], the possibility was studied of a further verification of formula φ on the structure K_N, provided that the formula K_N holds for the reduced structure K_{CN}. Homomorphism of paths in the structures K_N and K_{CN} and the following possible cases for the path states π' in the structure K_{CN} are determined: 1) when the sequence of states that make up the path π' contains only images of the states of structure K_N, which are not states of any areas of weak connectivity of Kripke structure K_N simulating the operation of the respective compound component, allocated in Petri net of the system under consideration; 2) when the path π' contains a state-encapsulents. In the first case, the homomorphism path π' of the structure K_N is bijective and for the states of these paths, performability (non-performability) of formula φ on the path π' implies performability (non-performability) of this formula on the path π . In the second case, the following options are possible: a) formula φ is not fulfilled on the path π' of structure K_{CN}. Then, obviously, the formula φ is not fulfilled on the path π of the structure K_N; b) formula φ is fulfilled on the path π' of structure K_{CN}. In this case, special consideration is required for state-encapsulents g'_i and their archetypes — states g_{i_k} of the structure K_N which are the states of one section of weak connectivity of Kripke structure K_N that simulates the operation of the respective compound component, allocated in detailed Petri net of the system under consideration.

However, given the presence of the same and similar parallel processes in the studied system, not all state-encapsulents and, respectively, not all sections of weak connectivity of Kripke structure K_N can be considered that simulate the functioning of the compound components. It suffices to investigate only one of their representatives with fewer identical and similar parallel processes. Such investigation contains the verification of the performability of formula φ for all paths of the given section of structure K_N which are subpaths of the path π, homomorphic image of the corresponding path π' of structure K_{CN}.

Checking algorithm for CTL-logic is proposed for formulas using the logical operations of negation, conjunction, disjunction ($\neg$, $\wedge$, $\vee$) and CTL-operators ◊◯, ◊●, ◊◐, as these logic operations and operators can be reasonably regarded as basic [2, 6]: any CTL-formula can be written using only given logical operations and CTL-operators.

Proposed in [6] the process of checking for CTL-logic formulas on a section of weak connectivity of Kripke structure K_N that simulates the operation of the respective compound component, allocated in detailed of Petri net of studied system, allows us to check not all formulas, but only the formulas recorded by base connections and operators. We want to get a universal opportunity to test the veracity of the formula φ or the possibility to affirm that in certain conditions in the sections of weak connectivity of structure K_N, that simulate the functioning of the relevant compound components allocated in the detailed Petri net N of the system under consideration, to establish the accuracy of the formula φ on the structure K_{CN} implies the accuracy of the formula on structure K_N.

3. Necessary and sufficient conditions for checking the accuracy of CTL-formula in a section of weak connectivity of Kripke structure simulating the operation of the respective compound component, allocated in detailed Petri net of the system under consideration

During verification of the performability of formula φ in the relevant section of weak connectivity of Kripke structure K_N, that simulates the operation of the compound component and meets its state-encapsulant g_i' of the structure K_{CN} in [6], the necessary sequence of actions was established for the inspection of all possible subformulae for formulas written using basic connectors and operators. Verification of $mutex$ and $fairness$ properties was considered separately — it is important for models with concurrency. At the same time, atomic statement φ and formulas $\neg\varphi$, $\varphi \vee \psi$, $\neg(\varphi \vee \psi)$, $\varphi \wedge \psi$, $\neg(\varphi \wedge \psi)$, $\Diamond \bigcirc \varphi$, $\Diamond \bullet \varphi$, $\Diamond◐(\varphi, \psi)$ were subject to testing. These checks were carried out in the states of paths π_k of sections π of sections of the structure K_N that respond to particular states-encapsulants g_i' of the structure K_{CN}. Such paths π_k are subpaths of corresponding path π of the structure K_N, whose image under the homomorphism h is the appropriate path π' of the structure K_{CN}.

1. Necessary conditions that should be possessed by sections of weak connectivity of the structure K_N, simulating the operation of the compound component of the Petri net of the studied system with concurrency, to establish the accuracy of CTL-formulas on the paths of these sections.

The condition of presence, on the section of weak connectivity of the structure K_N, simulating the operation of the compound component, the path π_k being the subpath of the path π of structure K_N, which homomorphic image is path π' of structure K_{CN}.

For this, there should not be dead states on the section under investigation of the structure K_N — the states, from which there are no transitions to other states. That can mean either constructing error both of the Kripke structure and the original Petri net model, or the incompleteness or inaccuracy of the original data. The existence of these troubles can be set on the stage of allocation of compound components in the constructed Petri model of the studied system. In this case, deadlocks and traps in sections of Petri nets, corresponding to chosen compound components, can be found by solving a system of logical equations, which describe these properties, or equivalent system of homogeneous linear Diophantine inequalities over the set $\{0, 1\}$ [10, 11].

For Petri net, non-empty set of places Q is called a) deadlock, if and only if ${}^{\bullet}Q \subseteq Q^{\bullet}$; b) trap, if and only if $Q^{\bullet} \subseteq {}^{\bullet}Q$.

In deadlock, all input transitions for set Q are its output transitions, resulting in none of them can not work if there is no tokens in Q. In trap, all output transitions for set of places Q are its input transitions, bringing non-decreasing number of tokens in the trap.

But not every deadlock leads to the dead transitions. According to theorem [11], Petri net of free choice is alive if and only if each deadlock of the net has trap labeled with initial marking. This fact for certain Petri net can also be established by writing logic equations and the equivalent system of linear homogeneous Diophantine inequalities over the set $\{0, 1\}$ [11].

Theorem 1. If there are no deadlock states in Kripke structure K_N of the studied system with concurrency, then for any section of weak connectivity of the structure K_N that simulates the operation of the compound component, allocated in the Petri net model of the studied system, there is always a path π_k, being a subpath π of the structure K_N, which homomorphic image is path π' of the structure K_{CN}.

The proof of Theorem 1 is based on established in [6] strong consistency [12] of reflection h with the same relations. Here we consider the relationship of transitions R and R', and the relationships of components χ_1 and χ'_1. Herewith Kripke structures K_N and K_{CN} are represented as one-type algebraic systems [13] $K_N = (G, R, \chi_1)$ and $K_{CN} = (G', R', \chi'_1)$. Then for states of homomorphic models $K_N = (G, R, \chi_1)$ and $K_{CN} = (G', R', \chi'_1)$ we can justify as follows:

1) for the two states g_1 and g_2 of the structure K_N, only one of them, g_1 or g_2 is the state of the section with weak connectivity of the structure K_N that simulates the operation of the respective compound components, allocated in detailed Petri net N of

the system under consideration, the following is performed;

$$(R'(h(g_1), h(g_2)) = T) \Longrightarrow \exists_G\, g'_1, g'_2\ ((h(g_k) = h(g'_k), k = 1, 2) \wedge (R(g'_1, g'_2) = T) \qquad (1);$$

2) for two states g_1 and g_2 of the structure K_N, which are the states of one section of weak connectivity of the structure K_N that simulates the operation of the respective compound component, allocated in the detailed Petri net N, the following is performed:

$$(\chi'_1(h(g_1), h(g_2)) = T) \Longrightarrow \exists_G\, g'_1, g'_2\ ((h(g_k) = h(g'_k), k = 1, 2) \wedge (R(g'_1, g'_2) = T) \qquad (2).$$

2. Sufficient conditions required for sections with weak connectivity of the structure K_N, simulating the operation of the compound components of the Petri nets of the studied system with concurrency, to determine the accuracy of CTL-formulas on the paths of these sections.

By definition of the Kripke structure, for the structure $K_{CN} = (G', R', \chi'_1)$ function $f' : G' \to B(P)$ marks each state $g' \in G'$ of the structure with set of atomic statements that are true in this state. This set is denoted by $lable(g)$. So the set of true in g'_i atomic statements — $lable(g'_i)$ is mapped with states-encapsulants g'_i of the structure K_{CN}.

Let φ is an atomic statement belonging to the set $lable(g'_i)$. In this case, φ is an atomic statement of state g'_i and $K_{CN}, g'_i \models \varphi$. Then, to establish the accuracy of CTL-formula on K_N by accuracy of this formula on K_{CN} it is sufficient to verify that φ the is atomic statement of the path π_k. Where path π_k — the path of section of weak connectivity of the structure K_N, which is encapsulated in the mapping h in the state g'_i, i.e. $K_{CN}, \pi_k \models \varphi$.

Let state-encapsulant formula g'_i of the structure K_{CN} is the formula $\Diamond \bigcirc \varphi$ or formula $\Diamond \bullet \varphi$, or formula $\Diamond ◐ (\varphi, \psi)$. Rules for the implementation of these formulas for the structure K_{CN} are as follows:

1). If g'_i of the structure K_{CN} holds formula $\Diamond \bigcirc \varphi$, then there is a condition g' in K_{CN}, in which there is a transition $((g'_i, g') \in R')$ from the state g'_i and such that $g'(\varphi) = 1$.

2). If in g'_i of the structure K_{CN} the formula $\Diamond \bullet \varphi$ is fulfilled, then in the structure K_{CN} from the state g'_i there is a path π' so that for any state g' of this path $g'(\varphi) = 1$ is executed.

3). If in g'_i of the structure K_{CN} the formula $\Diamond ◐ (\varphi, \psi)$ is fulfilled, then in the structure K_{CN} there is a path π' from state g'_i, and the state g' of path π', so that $g'(\psi) = 1$ and for any state g'' preceding state g' on this path, a condition $g''(\varphi) = 1$ is fulfilled.

Considering rules 1) - 3), and considering successions (1) and (2) from the proof of Theorem 1, one can prove, that in order to establish the accuracy of such CTL-formula on K_N by the accuracy of this formula on K_{CN}, it is sufficient to find such path π_k, which is a subpath of the path π, a homomorphic image of which is appropriate path π' of the structure K_{CN}, in states that the formula φ is satisfied, i.e. φ is a formula of the path π_k. So, theorem holds:

Theorem 2. Temporal CTL logic formula is true in a Kripke structure K_N, if it is true on Kripke structure K_{CN} that is homomorphic to it and the following implementations are true: 1) if the atomic statement φ is true in a state-encapsulant g'_i of the structure K_{CN}, then it must be true in all states of the path π_k of the section of the structure K_N that encapsulates in the state g'_i at the homomorphism of structures K_N and K_{CN}; 2) if formula $\varphi \vee \psi$ or $\Diamond \bigcirc \varphi$, or $\Diamond \bullet \varphi$, or $\Diamond◐(\varphi, \psi)$ is formula of state-encapsulant g'_i of the structure K_{CN}, then formula φ (or ψ only in case of formula $\varphi \vee \psi$) must be a formula of path π_k of the structure K_N; 3) if the formula $\neg\varphi$ is the formula of state-encapsulant g'_i of the structure K_{CN}, then formula $\neg\varphi$ should be the formula of path π_k of the structure K_N.

4. Conclusion

The article continues initiated in [5, 6] study on the possibilities of attracting apparatus of component Petri nets for the verification of parallel distributed systems using the automatic validation of the method $ModelChecking$, which involves the use of a semantic Kripke structure and apparatus of temporal logic CTL. Necessary and sufficient conditions are formulated which sectors of weak connectivity of Kripke structure of detailed Petri net of studied system must have to participate in the verification of CTL-formulas on Kripke structure, which encapsulates at homomorphism in the states of Kripke structure of component Petri net, that has significantly smaller sizes than original model.

References

1. *Clarke, E. and Grumberg, O. and Peled, D.* 1999. Model Checking. The MIT Press, 314 p.
2. *Mironov, A.* Program Verification by Model Checking. http://www.intsys.msu.ru/staff/mironov/modelchk.pdf
3. *Velder, S. and Shalyto, A.* 2007. On Verification of Simple Automation Programs Based on Model Checking Method. Informationno-Upavlyayuschie Systemy, 3, pp. 5–12.
4. *Mironov, A.* Mathematical Theory of Software Systems. http://www.intsys.msu.ru/staff/mironov/mthprogsys.pdf.
5. *Lukyanova, E.* 2014. Application of Component Petri Nets in the Tasks of Verification of Parallel Distributed Systems. Problemy programuvannya, 2–3, pp. 93–98.
6. *Lukyanova, E.* 2014. On Verifying Formulas of CTL-Logic on Kripke Models of Petri Net of Parallel Distributed System. Bionica Intellecta, 1 (82), pp. 29–35.
7. *Lukyanova, E.* 2011. On Component Analysis of Parallel Distributed Systems. TVIM, 2, pp. 71-81.

8. *Lukyanova, E.* 2012. The Structural Elements of a Petri Net Component. Problemy programuvannya, 2–3, pp. 25–32.
9. *Lukyanova, E.* 2013. On Bisimulational Equivalence of Detailed Petri Net and Its CN-model of the Investigated Parallel Distributed System. Visnyk of Kyiv Univ. im. Tarasa Shevchenka. Series: physico-matematical sciences, Special Issue.
10. *Murata, T.* 1989. Petri Nets: Properties, Analysis and Applications. In Proceedings of the IEEE, V. 77, 4, pp. 541–580.
11. *Kryvy, S.* 2007. Discrete Mathematics. Selected Problems. Kyiv: Kievo-Mogilyanska academia, 570 p.
12. *Gurov, S.* 2002. Elements of the Theory of Ordered Sets and Universal Algebra. Moscow: Publishing Department of the Faculty of Computational Mathematics and Cybernetics, 100 p.
13. *Maltsev, A.* 1970. Algebraic systems. Moscow: Nauka, 392 p.

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