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THE ALGORITHM FOR PRELIMINARY DISCRETE PATTERN RECOGNITION[1]

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Abstract

A method is proposed for a preliminary classification of binary described objects based on monotone separators. Constructed separators serves as a basis for bijective transformation of defining Boolean cube. The monotonizing transformation conserves a certain potential-type closeness function of vertices of Boolean cube which suggests to perform the classification of objects.

Introduction

Statement of problem. The problem of classification of binary described objects (all features have a value 0 or 1), stands out as a specific one among pattern recognition tasks. The importance of this type of tasks is due to the following circumstances. First, they are encountered in practice. Second, and most important, replacing continuous features by binary ones negligibly affects the recognition accuracy since data used in descriptive sciences, e. g. medicine, geology, etc, are usually presented with a moderate precision. From this viewpoint such replacement usually causes no significant deterioration of solution.

Last achievements analysis. A common heuristic approach to the pattern recognition of an object described by binary information includes the following stages. At the first stage a partial Boolean function (BF) f which takes value 1(0) on standard training sample of the first (second) classes is formed. This BF is defined on a n-dimensional Boolean cube E_n, where n is the number of objects features. In this approach the most interesting part of a recognition algorithm lies in selecting one or another rule for extending a determination f of the whole BC or its part. Furthermore, for each new object the predetermined value of f is clearing up. This value determines the object reference to one of the two classes.

The choice of the above-mentioned extension of a definition rule conceptually implies a specification of separators of subsets of the BC vertices A and B which are the calibration samples from two classes of objects in the binary feature space. *Desideratum* in this approach is the choice in a manner best of this extension. This rule may be determined by some closeness function (metric) on the BC between the given subsets and vertices outside these subsets. This metric is in its turn conventionally specified as a potential function. Thereupon a indeterminate value of f is extended to 1(0) if the value of a potential function is positive (negative) at the presented vertex of BC and the object is

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assigned to the first (second) class. Zero value of the potential can be considered as a classification algorithm failure.

This paper has intent to propose accomplishing subsets separation by a monotone BF. More specifically, we shall describe transformation of the BC that allows approximating two given subset of its vertices by two monotone BF. This transformation effectively solves the problem of constructing monotone separators of given subsets. The choice of monotone functions as approximating ones is motivated by the simple structure of this class of functions that results in uniqueness of its extremal normal forms and simplifies a technical realization. This transformation is obviously not a Hamming transformation since ia completely conserves the structure of defining the BC and does not extend the class of possible separators. Therefore, a weaker condition than a Hamming isometry should be imposed on the required transformation. Such condition is bijectively: we shall show that bijective transformations change the structure of the feature space so that monotone separators exist, yet the change is mild enough to produce a conservative metric which is natural in a certain sense. The paper describes a transformation of the BC, approximating the monotone BF, and a potential function having desired properties.

1. Basic notation. Statement of the problem of constructing monotone separators

The BC vertices (vectors) are denoted $\tilde{\alpha} = (\alpha_1, \dots, \alpha_n)$. The symbols $\tilde{1}$ and $\tilde{0}$ denote vectors $(1, \dots, 1)$ and $(0, \dots, 0)$ respectively. $1(\tilde{\alpha}) = \{i | \alpha_i = 1, i = \overline{1,n}\}$, $0(\tilde{\beta}) = \{i | \beta_i = 1, i = \overline{1,n}\}$.

A set of units and zeros of the BF f is denoted by N_f^1 and N_f^0, $N_f^- = E^n \backslash (N_f^1 \cup N_f^0)$; if $N^- \neq \otimes$ BF f is a partial function. A set of the monotone BF of n variables will be denoted by M_n. The function «*mod* 2 sum» is denoted $\oplus$, the conjunction — by $\bigwedge$, the identity — by $\equiv$, the implication — by $\rightarrow$, the negation — by $\neg$. Notation x^α means x or $\neg x$ if $\alpha = 0$ or $\alpha = 1$ respectively.

The transformation $F(x) : E^n \to E^n$ is tuple of BF $\langle \varphi_i(\tilde{x}) \rangle, \tilde{x} \in E^n, i = \overline{1,n}$, such that $F(x_1, \dots, x_n) = (x_1 \oplus \varphi(x_1), \dots, x_n \oplus \varphi(x_n))$ (subsequently we shall consider bijective transformations of the BC only). Usually we will omit the variable x and write F. The result of F action on the vertex $\tilde{\alpha}$ and the set of vertices G will be denoted by $F(\tilde{\alpha})$ and $F(G)$, respectively.

The transformation F is called *monotonizing* if $f(F(E^n)) \in M_n$. The sets $\tilde{\alpha}^{\triangle} = \{\tilde{\beta} | \tilde{\beta} \geqslant \tilde{\alpha}\}$ and $\tilde{\alpha}^{\nabla} = \{\tilde{\beta} | \tilde{\beta} \leqslant \tilde{\alpha}\}$ are lower and upper cone respectively of $\tilde{\alpha} \in E^n$. A unit vector of the BF f is a *lower unit* of f if its lower cone contains no other unit vectors of f. Let a set of lower units of the BF f be denoted by $LU(f)$. A zero vector BF f is an *upper zero* of f if its upper cone contains no other zero vectors of f. Let a set of upper zeros of the BF f be denoted by $UZ(f)$. BFs f_φ^0 and f_φ^1 relating with BF φ by conditions $LU(f_\varphi^0) = LU(\varphi)$, $UZ(f_\varphi^1) = UZ(\varphi)$, f_φ^0, $f_\varphi^1 \in M_n$ are called *majorant* and *minorant* of the φ. The *antimonotonicity* rank of the BF f denoted by $Ram(f)$ is a total number of zeros $\tilde{\beta}$ so that there exist a lower unit $\tilde{\alpha}$ less then $\tilde{\beta}$ and the number of unit vectors $\tilde{\alpha}$ so that there exist an upper zero $\tilde{\beta}$ greater than $\tilde{\alpha}$. If the antimonotonicity rank of partial BF is zero, we shall call it a partial monotone BF. Otherwise, the intersection of a set of unit vectors of f^0_φ and a set of zero vectors of f^1_φ will be called the *gap region of* φ.

Suppose that nonintersecting subsets A and B in E^n define the BF $f(E^n)$ on E^n, with A defining a set of unit vectors and B a set of zero vectors of f. It is required to find a monotonizing bijection W_f of BC E^n. In other words, we need to find a bijective transformation of E^n so that on $W_f(E^n)$ we have $f^0_f(\tilde{x}) \to f^1_f(\tilde{x})$, $\forall \tilde{x} \in E^n$. We naturally augment this problem formulation with requirement of minimum complexity of monotonizing bijection (defined below).

2. Solution of the problem

We shall represent the required monotonizing bijection W_f on the BC as a composition of a Hamming-isometric transformation (motion) U_f and Hamming-nonisometric transformation V_f (i. e. $W_f = U_f * V_f$, where $*$ is the symbol of composition of functions).

The motion D_γ with a polarization vector $\tilde{\gamma}$ is the transformation $F(\tilde{x}) = (x_1 \oplus \gamma_1, ..., x_n \oplus \gamma_n)$. Clearly the motions describe all possible Hamming-isometric transformation of the BC corresponding to the inversions of the variables. Now we shall find a Hamming-nonisometric part U_f of the required transformation W_f, assuming that some motion $D_{\tilde{\gamma}} = V_f$ has been performed (and this means we work on cube $D_{\tilde{\gamma}}(E^n)$.

A transposition of vectors $\tilde{\alpha}$ and $\tilde{\beta}$ of the BC T is bijection $T_{\tilde{\alpha},\tilde{\beta}} = \langle \tau_i(\tilde{x}) \rangle$ defined by the function

$$\tau_i(\tilde{x}) = \begin{cases} 0, & \text{if } \alpha_i = \beta_i, \quad i = \overline{1,n}, \\ \bigwedge\limits_{j=1}^{n} x_j^{\alpha_j} \oplus \bigwedge\limits_{j=1}^{n} x_j^{\beta_j} & \text{otherwise}, \quad i = \overline{1,n}. \end{cases}$$

Informally, a transposition indicates that the BC $T_{\tilde{\alpha}\tilde{\beta}}(E^n)$ is obtained from the BC E^n interchanging the vertices $\tilde{\alpha}$ and $\tilde{\beta}$. Put $K_{\tilde{\alpha}}(\tilde{x}) = \bigwedge\limits_{j=1}^{n} x_j^{\alpha_j}$. Them $\tau_i(\tilde{x}) = K_{\tilde{\alpha}}(\tilde{x}) \oplus K_{\tilde{\alpha}}(\tilde{x})$ for $i \in 0(\tilde{\alpha} \equiv \tilde{\beta})$ and $\tau_i(\tilde{x}) = 0$ for $i \in 1(\tilde{\alpha} \equiv \tilde{\beta})$. A transposition of $\tilde{\alpha}$ and $\tilde{\beta}$ is admissible for the BF f if one vertex is a lower unit and another is an upper zero of f and $\tilde{\alpha}$ and $\tilde{\beta}$ is comparable.

If f is a monotone BF, then U_f is the identity transformation. Otherwise ($Ram(f) \neq 0$), there are vectors $\tilde{\alpha}$ and $\tilde{\beta}$ so that $T_{\tilde{\alpha},\tilde{\beta}}$ is an admissible transformation. Let us perform this transformation. The antimonotonicity rank of f will decrease by less then 2 since at least two vectors are removed from the gap region of f. If antimonotonocity rank of f defined on the transformed BC is non-zero, then we again find a pair of vertices forming an admissible transposition, and so on. The process obviously terminates after at least $Ram(f)/2$ steps. A collection of all transpositions performed is the required monotonizing bijection.

The above-described algorithm leads to the following conclusions:

1. a monotonizing bijection always exists;
2. for a given BF f there exists in general a whole get of bijections that monotonize f (we call them admissible) that have different sequences of transposition pairs; the chice of one pair may exclude the choice of another pair;
3. a mutual distance between vertices not taking part in transpositions is conserved.

Suppose that this algorithm has produced a sequence of pairs $(\tilde{\alpha}^i, \tilde{\beta}^i), i = \overline{1,m}$, that corresponds to the chosen transposition. It can be shown that the tuple of functions $\langle u_j(\tilde{x})\rangle$ specifying a monotonizing bijection is defined by

$$u_j(\tilde{x}) = \bigoplus_{i:\, \alpha^i_j \neq \beta^i_j} \left(K_{\tilde{\alpha}^i}(\tilde{x}) \oplus K_{\tilde{\beta}^i}(\tilde{x})\right), \quad j = \overline{1,n}.$$

A complexity of nonisometric bijection U_f is the number m of its transpositions (note that under this definition an equivalent bijection may have different complexity). We shall describe an algorithm for selecting transposition pairs of the U_f. Let us consider the set Z of all comparable pairs $\{(\tilde{\alpha}, \tilde{\beta})\}$ where $\tilde{\alpha}$ and $\tilde{\beta}$ compose an admissible transposition of f. Let us compile a bipartite graph Γ. The «parts» of the vertex set consist of unit and zero vectors of f from Z respectively. The pairs $\{(\tilde{\alpha}, \tilde{\beta})\}$ from Z are the edges of Γ. Let us chose in Γ a pair of adjacent vertices with a maximum sum of degrees and mark it. Remove all the marked vertices from Γ, their incident edges, and also the isolated vertices that appear in process. Apply the same rule to mark new vertices, and so on until the empty graph is obtained. The sequence of marked vertices obtained in the process obviously defines the transpositions of the monotonizing bijection U_f. It can be shown that the problem of minimizing the number of transpositions reduces to the shortest covering problem and the greedy algorithm described above corresponds to a gradient method of solving this problem.

The problem of determinations of $\tilde{\gamma}$ for motion $D_{\tilde{\gamma}} = V_f$ performed before U_f is not easy then NP-complete lack-tautology problem. In [1] the approximate method of determinations of $\tilde{\gamma}$ is described.

3. Potential and classification

We call the upper $G^{\triangle}$ (lower G^{∇}) cone of the set $G \in E^n$ the sum of all upper $\tilde{\gamma}^{\triangle}$ (lower $\tilde{\gamma}^{\nabla}$) cones of all the elements $\tilde{\gamma} \in G$.

Let us define the upper $US_f(\tilde{\omega})$ and lower $LS_f(\tilde{\omega})$ sets for arbitrary $\tilde{\omega} \in E^n$ with reference to BF f. The $US_f(\tilde{\omega})$ consists of zero vectors of f greater than $\tilde{\omega}$; $LS_f(\tilde{\omega})$

consists of unit vectors of f less than $\tilde{\omega}$. Using $US_f(\tilde{\omega})$ and $LS_f(\tilde{\omega})$ we define the upper value $u_f(\tilde{\omega})$ and lower value $l_f(\tilde{\omega})$ for $\tilde{\omega} \in E^n$.

To define $u_f(\tilde{\omega})$, construct the set $LS'_f(\tilde{\omega})$ which combines the unit vectors contained in $US_f^{\nabla}(\tilde{\omega})$ (note that $LS_f(\tilde{\omega}) \subseteq LS'_f(\tilde{\omega})$). Now, if for $\tilde{\beta} \in US_f(\tilde{\omega})$ there exists $\tilde{\alpha} \in LS'_f(\tilde{\omega})$ that $\tilde{\alpha} < \tilde{\beta}$, then eliminating these vertices from the corresponding sets, we say that an operation of mutual elements has been performed. We apply this mutual removing operation as long as possible. Consider the result of this procedure for the upper set with all possible choices of the removed pair, i. e. irredundant subsets $US_f(\tilde{\omega})$ to which the mutual removing operation is inapplicable. Let $u_f(\tilde{\omega})$ be equal to the least cardinality among the irredundant subsets.

Similarly to define $l_f(\tilde{\omega})$, let us construct the set $US'_f(\tilde{\omega})$ that consist of the zero vectors of f contained in the upper cone of the lower set for $\tilde{\omega}$ and is an extension of $US_f(\tilde{\omega})$. We reduce the set $LS_f(\tilde{\omega})$ by removing from it and from $US_f(\tilde{\omega})$ those elements $\tilde{\alpha}$ and $\tilde{\beta}$ (respectively) for $\tilde{\alpha} < \tilde{\beta}$. Value $l_f(\tilde{\omega})$ equals to the least cardinality among the irredundant subsets generated from $LS_f(\tilde{\omega})$ by this removal operation with all possible choices of the removed pairs. It follows from the definition of upper and lower values that they do not exceed the cardinalities of corresponding sets and at least one of them equals 0.

For an arbitrary element $\tilde{\omega}$ of BC E^n with BF f, we define the potential function $P_f(\tilde{\omega})$,

$$P_f(\tilde{\omega}) = \begin{cases} +p, & if \tilde{\omega} \in N_f^1, \\ -p, & if \tilde{\omega} \in N_f^0, \\ l_f(\tilde{\omega}) - u_f(\tilde{\omega}), & if \tilde{\omega} \in N_f^-, \end{cases}$$

where p is an arbitrary natural number.

The followings propositions may be proved [2].

1. If a vertex $\tilde{\alpha} \in N_f^1$ exists so that the upper cone $\tilde{\alpha}$ dose not contain zeros of f and $\tilde{\alpha} < \tilde{\omega}$ then $P_f(\tilde{\omega}) > 0$.
2. If a vertex $\tilde{\beta} \in N_f^0$ exists so that the lower cone $\tilde{\beta}$ dose not contain units of f and $\tilde{\omega} < \tilde{\beta}$ then $P_f(\tilde{\omega}) < 0$.
3. For the BF $f(E^n)$ and an arbitrary $\tilde{\omega} \in E^n$ the inequality $P_f(\tilde{\omega}) > 0$ implies $f_f^0(\tilde{\omega}) = 1$ and $P_f(\tilde{\omega}) < 0$ implies $f_f^0(\tilde{\omega}) = 0$.
4. If f is partial monotone[2], then the word «implies» in the above proposition may be replaced by «is equal to».
5. $U_f(\tilde{x})$ conserves the sign of the potential $P_f(\tilde{\omega})$.
6. $U_f(\tilde{x})$ conserves the zero value of the potential for the vertices in $C_f = \{\tilde{\omega} \in E^n | (\neg f_f^0(\tilde{x}) \wedge f_f^1(\tilde{x})) = 1\}$.

[2]We call BF f partial monotone if $\bigcup\limits_{\tilde{\alpha} \in N_f^1} \tilde{\alpha}^{\triangle} \bigcap \bigcup\limits_{\tilde{\beta} \in N_f^0} = \oslash$.

The above-listed propositions suggest that a sign of the potential may be used for classification of vertices of BC. Let the BF f take the value 1 (0) on a standard training sample of the first A (second B) classes (i. e. $N_f^1 = A$ and $N_f^0 = B$) and $W_f = U_f * V_f$ is constructed as pointed above. Then assign $\tilde{\omega}$ to A if $P_f(W_f(\tilde{\omega})) > 0$ and to B if $P_f(W_f(\tilde{\omega})) < 0$. Zero value of the potential indicates the algorithm failure to perform the preliminary classification. Note that this classification for $P_f(\tilde{\omega}) \neq 0$ is entirely natural. A failure of preliminary classification for $\tilde{\omega} \in C_f$ is also natural (insufficient information) and allow u to continue with other classification algorithms. Moreover, note that by a proposition (4) potential $P_f(W_f(\tilde{\omega}))$ is easily evaluated from the easily found majorant and minorant of BF f.

The method may obviously be generalized in the problem of classification with a number of classes. Clearly offered algorithm has infinite VCD.

Conclusion

The maim result of the paper is the offered algorithm for preliminary recognition. This algorithm based on the approach using potential witch is original for discrete patterns. The results of the work may be used in solving of various practical problems. *Suggested technique* may be used for solving others discrete tasks.

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